LONGITUDINAL SPACE-CHARGE FORCES AT TRANSITION--SCALING RELATIONS

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Hereward and Sørenssen, in a series of CERN reports,[1, 2, 3, 4] consider the effects of longitudinal space-charge forces in a synchrotron, and in particular the balance between these forces and the rf focusing forces. The space charge tends to distort the beam's phase distribution, leading to mismatch and blowup after transition. In (3) and (4) it is shown that this can be compensated by a device known as the "triple switch", but that this compensation breaks down when the space charge is too intense. The CERN PS, operating at $\sim 10^{12}$ protons per pulse, exhibits the effect in a range where compensation is possible and has been reasonably well achieved.

Quantitatively, the measure of the effect is given by a quantity known as $\eta_o(0)$, defined as follows: $\eta_o(0)$ = ratio of space-charge defocusing force to rf restoring force at transition energy, with the space-charge force calculated from the bunch density that would obtain at transition energy if space charge were neglected; i.e., calculated by the "classical" (low density) theory of adiabatic behavior of synchrotron oscillations between injection and transition.

Hereward (2) derives the following expression for $\eta_o(0)$:

$$\eta_o(0) = \frac{3}{2}\,\frac{r_p}{R}\,\frac{2\pi h\, mc^2}{\gamma^2\, eV|\cos\varphi_s|}\,\frac{N g_o}{\hat{\theta}^3} \tag{1}$$

where g_o is a geometrical factor, $\hat{\theta}$ = half-length of bunch in rf radians, and the other symbols have their conventional meaning, all evaluated at transition energy.

To compare different accelerator designs, we must obtain an expression for $\hat{\theta}$. Hereward (2) shows:

$$\hat{\theta} = 0.49 \left(\frac{S}{K_1}\right)^{1/2} \tag{2}$$

where S is the canonical phase-space area occupied by one bucket, and

$$K_1 = \text{const} \times R \left(\frac{\gamma^4}{h^4} \; eV \frac{\cos^2 \varphi_s}{\sin \varphi_s}\right)^{1/3} \quad \text{(at transition)} . \tag{3}$$

Finally, S may be calculated on the assumption that the bucket is filled at injection, giving

$$S = \text{const} \times \alpha (\varphi_s) R \left(\frac{\gamma}{|\eta|}\right)^{1/2} (eV)^{1/2} h^{-3/2} \tag{4}$$

where all parameters are evaluated at injection; α is a certain function of φ_s (tabulated in (2)); $\eta = \frac{1}{\gamma^2} - \frac{1}{\gamma_{tr}^2}$.

Continuing these formulas, and keeping φ_s constant, we find that the space-charge parameter $\eta_o(0)$scales as

$$\frac{N}{R} \frac{h^{5/4}}{V_t^{1/2} V_i^{3/4}} \left(\frac{\eta_i}{\gamma_i}\right)^{3/4} .$$

Note, in particular, that the dependence on the transition energy cancels out; at higher transition energy, the factor γ^{-2} in (1) is just cancelled by the fact that the bunching $\hat{\theta}$ becomes tighter.

If injection into the synchrotron is from a booster of smaller radius, with matching bunches, then S [Eq. (4)] has to be calculated for the booster. If the ratio of radii (synchrotron to booster) is $R_S/R_B = h_S/h_B = M$, the scaling law becomes

$$\frac{N}{R} \quad \frac{h^{5/4}}{(M\,V_i)^{3/4}\;V_t^{1/2}} \quad \left(\frac{\eta_i}{\gamma_i}\right)^{3/4} .$$

Table I shows comparative parameters for some present and projected synchrotrons. It is seen that the NAL booster will be no worse than the CPS is now; in the NAL main synchrotron the problem will be somewhat more severe. The BNL AGS is - fortuitously - much better off than the CPS, and the problem will be most severe in the CPS with future improvements.

Sørenssen (4) shows that the distortion can be compensated by the "triple switch" mechanism (3) when η_o (0) $\leqslant$ 6. Therefore all cases considered here are amenable to compensation.

REFERENCES

1. A. Sørenssen and H. G. Hereward, MPS/Int. MU/EP 66-1.
2. H. G. Hereward, MPS/DL Int. 66-3.
3. A. Sørenssen, MPS/Int. MU/EP 66-2.
4. A. Sørenssen, MPS/Int. MU/EP 67-2.

TABLE I

	N	R (m)	h	M	V_i (MV/turn)	V_t	$\left(\frac{\eta_i}{\gamma_i}\right)$	$\eta_o^{(0)}$
CPS (now)	10^{12}	100	20	1	0.05	0.05	0.9	1.3
CPS (future)	10^{13}	100	20	4	0.025	0.1	0.9	5.6
BNL-AGS	2×10^{12}	128	12	1	0.1	0.1	0.9	0.45
BNL-AGS (Improved)	10^{13}	128	12	1	0.2	0.2	0.7	1.6
NAL Booster	2.3×10^{12}	75	75	1	0.2	1.0	0.7	1.3
NAL Synchrotron	3×10^{13}	1000	1000	13.3	0.2	4.5	0.7	2.15